A simple Kalman filter used in Dynare

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1 State–space representation

$$\begin{aligned} y_t &= Z_t a_t + \eta_t \\ a_t &= T a_{t-1} + \epsilon_t \\ E(\eta_t \eta_t') &= H \\ E(\epsilon_t \epsilon_t') &= Q \end{aligned}$$

Z_t and H_t have time subscript to take into account the possibility of missing observations.

2 Kalman filter

For $t = 1, \ldots, T$

$$\begin{aligned} a_{t|t-1} &= T a_{t-1|t-1} \\ P_{t|t-1} &= T P_{t-1|t-1} T' + Q \\ v_t &= y_t - Z a_{t|t-1} \\ F_t &= Z_t P_{t|t-1} Z_t' + H_t \\ K_t &= P_{t|t-1} Z' F_t^{-1} \\ a_{t|t} &= a_{t|t-1} + K_t v_t \\ P_{t|t} &= (I - K_t Z) P_{t|t-1} \end{aligned}$$

- T is upper quasi-triangular
- There two operations involving T:
  - $T a_{t-1|t-1}$ where $a_{t-1|t-1}$ is a vector
  - $T P_{t-1|t-1} T'$ where $P_{t-1|t-1}$ is a symmetric matrix

  both should be optimized.
- It is necessary to find the right order and the right factorization of these expressions to minimize the number of operations.

- For likelihood computation none of these matrices and vectors need to be saved.
- There are no constant terms, because, in Dynare, we are removing them before entering the filter (at least we should).