Proposal for a Dynare interface to SWZ Markov Switching code

M. Juillard

17 January 2010, Version 1.6

1 SVAR identification

1. well known schemes are identified by option. Currently:

```
svar_identification(upper_choleksy);
svar_identification(lower_choleksy);
```

2. more sophisticated schemes are described by listing exculsion restrictions on A_0 or A_+:

```
svar_identification;
exclusion lag 0;
equation 1, y, pi;
equation 2, pi, r;
exclusion lag 1;
equation 1, y, pi;
equation 2, pi, r;
end;
```

This generates

```
M_.svar_exclusions{lag}(2xn matrix with equation numbers and restricted variable IDs)
```

The SVAR model is written as

$$y_t\, A_0(s_t) = x_t\, A_+(s_t) + \epsilon_t\, \Xi^{-1}(s_t),$$

where y_t is an $n \times 1$ vector of endogenous variables, x_t is a vector of all lagged variables plus the constant term, the state s_t follows a Markov process, $\Xi^{-1}(s_t)$ is a diagonal matrix in which the diagonal elements represent the state-dependent shock variances, and ϵ_t has a Gaussian distribution with mean 0 and variance matrix I. The order of x_t follows this convention. The ordering of variables in each equation has the following convention. The n variables at the first lag are order first, followed by the n variables at the second lag, and so on. The last variable is the constant term.

2 Restrictions on SVAR Markov-switching processes

SWZ introduce a restriction on the lagged coefficients in the SVAR. It can be called by `svar_restriction(SWZ)`. The restriction is essential to overparameterization by preventing an *excessive* number of parameters from changing from one state to another. The restrictions take the following form:

$$A_+(i,j,\ell,s_t) = g(i,j,\ell)\delta(i,j,s_t),$$

where i stands for the i^{th} variable, j for the j^{th} equation, and ℓ for the ℓ^{th} lag. This expression indicates that the coefficients at the first lag may change with state but the coefficients at other lags in each state are proportional those at the first lag in that state.

3 Prior specification on SVAR coefficients

The Sims and Zha (1998) prior applies to $A_0(s_t)$ for all s_t and $g(i,j,\ell)$ (in the original article by Sims and Zha (1998), the hyperparameter λ_2 is always set to 1). There are six hyperparameters controlling the tightness of this prior:

1. μ_1 controls overall tightness of the random walk prior (same as λ_0 in Sims and Zha (1998)).
2. μ_2 controls relative tightness of the random walk prior on the lagged coefficients (same as λ_1 in Sims and Zha (1998)).
3. μ_3 controls relative tightness of the random walk prior on the constant term (same as λ_4 in Sims and Zha (1998)).
4. μ_4 controls tightness of the prior that dampens the erratic sampling effects on lag coefficients (lag decay) (same as λ_3 in Sims and Zha (1998)).
5. μ_5 controls weight on the sum of coefficients in each equation through n dummy observations excluding the constant term. This component of the prior expresses belief about unit roots (same as μ_5 in Sims and Zha (1998)).
6. μ_6 controls weight on a single dummy initial observation including the constant term. This component of the prior expresses belief in cointegration relationships (up to $n-1$) and stationarity (same as μ_6 in Sims and Zha (1998)).

Note that while smaller values of μ_i for $i = 1,\ldots,4$ means a tighter random walk prior, larger values of μ_i for $i = 5,6$ means a tighter prior on unit roots and cointegration. We provide the following benchmark values of these hyperparameters, although one should vary the values for sensitivity check. For quarterly data, Sims and Zha (1998) suggest $\mu_1 = 1, \mu_2 = 1, \mu_3 = 0.1, \mu_4 = 1, \mu_5 = 1,$

and $\mu_6 = 1$. For monthly data, Sims and Zha (2006) suggest $\mu_1 = 0.57, \mu_2 = 0.13, \mu_3 = 0.1, \mu_4 = 1.2, \mu_5 = 10$, and $\mu_6 = 10$.

The prior on $\delta(i, j, s_t)$ for each i, j and s_t is a normal distribution with mean 0 and standard deviation 50. This prior is diffuse enough to allow for the possibility that VAR coefficients can have extremely large values for some states.

The prior on each element of the diagonal of $\Xi^2(s_t)$ (denoted as Zeta in our output file) is a gamma distribution, represented by Gamma$(\bar{\alpha}, \bar{\beta})$ with $\bar{\alpha} = 1$ and $\bar{\beta} = 1$.

4 Prior specification on Markov Switching processes

1. Priors on Marov Switching processes are specified through average duration of each state `markov_switching(chain=i,state=j,duration=d)` specifies that state j in chain i last on average d periods. Alternatively, if all the states have the same average duration, it is possible to simply declare the number of states in the chain with option `number_of_states` Example:

```
markov_switching(chain=1,state=1,duration=3);
markov_switching(chain=1,state=2,duration=0.5);
markov_switching(chain=2,state=1,duration=1);
markov_switching(chain=2,state=2,duration=4.5);
markov_switching(chain=2,state=3,duration=2);
markov_switching(chain=3,number_of_states=3,duration=2.5);
```

2. Transition matrix specification (for future use)

```
ms_chain(1) = [0.25*a d 0; 0.25*b e 0; . . 1];
ms_chain(2) = [a 0; . .];
```

The sum of the column must sum to 1. The dot (.) represents the complement to 1.

5 Associating Markov processes with coefficient matrices

1. default in the case of one chain: all coefficient matrices change

2. specific matrices are linked to specific chains:

```
svar(coefficients,chain=1);
svar(variances,chain=1);
svar(constants,chain=2);
```

3. specific equations are linked to specific chains:

```
svar(coefficients,equation=1,chain=2);
svar(variances,equation=3,chain=1);
svar(constants,equation=3,chain=1);
```